SOME CURRENT RESEARCH PROBLEMS IN AERONAUTICS

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1. Introduction

The influence and spirit of Ludwig Prandtl are indelibly impressed on the present generation of aeronautical scientists and engineers. His concepts have been and still are the foundation stones of our understanding of fluid mechanics and of engineering progress in aeronautics. He was an inspired and creative research worker who opened up new areas of theory and experiment. He was far ahead of his time. The overwhelming importance of his work was not appreciated until the aeronautical industry reached a relatively advanced state of engineering development. He studied supersonic flow [1] more than ten years before the first flight of an airplane at supersonic speeds. The boundary layer concept was started twenty years before the first measurements of velocity distribution were made. At that time only three laboratories were interested in the subject, namely Göttingen, Aachen, and Delft. There were about 20 papers on the boundary layer in the published literature of the 20 years from 1904 to 1924. Today there are as many papers every 60 days.

A year ago in the first Ludwig Prandtl Memorial Lecture Albert Betz reviewed in some detail the brilliant pioneering researches of Prandtl in fluid mechanics. In the light of this review Betz sought to determine the factors "on which the progress of our knowledge principally depends and what one ought to do to effectively further this progress". He described some of the effects of the tremendous growth of aeronautical research and development in inhibiting the activity of individual creative workers of the rare Prandtl type. He concluded that the best way of honoring Prandtl is to protect the valuable heritage which he has left to us and to strive to follow his example in attacking the problems confronting us.

This year you have given me the great privilege and honor of giving the second Ludwig Prandtl Memorial Lecture. I shall never forget the first visit of this great but kindly and modest man to our laboratory at the National Bureau of Standards. We were a group of relatively young men trying to follow in his footsteps to gain a better understanding of wind tunnel turbulence and of transition of the flow in the boundary layer from laminar to turbulent. His evident interest and words of encouragement were highly prized by our group. We were inspired to greater effort.

Today I am a member of a much larger team, immersed in that vast technology which, as Betz remarked, has risen above our heads and is sometimes reminiscent of the Tower of Babel. The National Advisory Committee for Aeronautics (NACA) is provided with large and expensive wind tunnels, high speed computing machines, and other equipment which do make a difficult environment for the gifted individual research worker. As I write these lines, NACA is approaching a metamorphosis into NASA, National Aeronautics and Space Agency, to undertake still larger responsibilities as the exploration of space begins. Nevertheless, there is a serious and sustained effort to approach the problems of our day in the spirit of Prandtl, to set forth simple conceptual models which include the essential physical processes and to make experiments to verify the utility and limitations of the model. Even in a large agency the human brain is the most important tool and the greatest possible freedom of the research worker the most essential factor in the laboratory environment.

Even in the NACA we have been able to provide opportunities for such men as A. Busemann and R. T. Jones, whom many of you know, as well as many others who have shown that they deserve the privilege to work on the problems which interest them most and to provide adequate support, so long as the problems lie within the field of interest of NACA.

In addition to the architects of the future such as Prandtl, progress in aeronautics requires the cooperative and creative effort of a host of workers, many organized into teams to exploit new concepts in their application to specific technical areas. The task of NACA to "study the scientific problems of flight with a view to their practical solution" or in different words, to advance aeronautical science and technology, has been attacked by combining the efforts of individuals and teams to advance knowledge on a broad front. I could think of no better way to honor Prandtl than to discuss some examples of current problems investigated by NACA in the broader framework of certain assumptions on which our research activities are based. Therefore I asked many of my colleagues to give me illustrations of research problems attacked in the spirit of Prandtl, i.e., analysis of the problem into its essential physical features and the use of a simplified mathematical and experimental approach leading to an understanding of fundamentals. I am greatly indebted to my associates for their collaboration.

2. Looking to the Future

In the NACA we have sought to obtain a fruitful interchange between research scientists and aircraft, missile, and engine designers. On the one hand the self-generated research work which aims to explore or understand often suggests pathways for engineering design studies in new directions. On the other a reasonable forecast of trends suggests new problem areas. A research agency must continually look to the future and try to anticipate development.

Fortunately the prediction of things to come does not have to be very precise. Research has a way of being applicable to a wide variety of specific vehicles; aerodynamic knowledge is applicable to airplanes, missiles, and returning satellites and, if sufficiently fundamental, to other fields of engineering as well as aeronautical.

In determining the research areas to which NACA will devote its resources, we assume today that there will be very low speed vertical or short take-off and landing aircraft, helicopters, water-based aircraft, supersonic Mach 3 fighters and bombers, hypersonic aircraft of the boost-glide variety, hypersonic anti-missile missiles, unmanned and manned satellites, and eventually interplanetary space craft. We assume that chemical and nuclear air breathing and rocket engines will be needed for propulsion. The practical development of such vehicles will obviously require knowledge in the fields of subsonic aerodynamics; hydrodynamics; transonic, supersonic, and hypersonic aerodynamics; low density aerodynamics; aerodynamic heating; combustion; cooling; high temperature structures; flow of ionized air; etc. Basic research is needed on the essential phenomena and also applied research on configurations thought to be of possible interest. Within such general guide lines and with some exposure to the current design thinking, individuals and groups can find stimulating and challenging problems. The following sections describe a few of these problems.

3. Application of Prandtl's Concept of Acceleration Potential

Everyone who is familiar with aerodynamic theory is aware of Prandtl's lifting line concepts and the important roles they played in the theory of high-aspect-ratio wings and early airplane developments. In 1936 [1] he introduced the less well known concept of acceleration potential, soon found especially useful for low-aspect-ratio wings.

This concept was first put to use by W. Kinner [2], a student of Dr. Prandtl, in a noteworthy treatment of the problem of determining the steady-state lift distribution on a circular plan form. Kinner's work was later extended by Krienes [3], to include the surface of elliptic plan form, and by Schade [4], to include a circular plan form with oscillating downwash conditions. Although the circle and ellipse are the only finite plan forms for which the lift distribution has been explicitly determined, the concept of acceleration potential opened a way to the development of reliable approximate methods for treating lifting surfaces of arbitrary plan form with either steady or time-dependent downwash conditions. Reliable methods for treating the steady case have for sometime been developed, for example by Multhopp [5], and by Falkner [6], but there has been a continuously growing need for a reliable method of treating the time dependent downwash cases. Such downwash conditions are inherent in the thin, low-aspect-ratio, flexible wings of many current and future aircraft, and a knowledge of the resulting aerodynamic forces is necessary in considerations of various aeroelastic problems such as flutter.

Prompted by this need for knowledge of the aerodynamics for unsteady wings, a general method, based on the acceleration potential, for treating an oscillating surface at either subsonic or supersonic speeds has recently been developed by C. E. Watkins, H. L. Runyan, and D. S. Woolston [7], at the NACA Langley Laboratory. The next few paragraphs give a brief description of the method and present some illustrative applications.

From the standpoint of an acceleration potential the boundary value problem for the pressure distribution on any surface can be readily reduced to an integral equation that relates the distribution of pressure on the surface to the distribution of downwash at the surface. In the case of an oscillating surface the integral equation is

$$\overline{w}(x, y) = \frac{1}{4\pi\rho V} \int_S\int \overline{\Delta p}(\xi, \eta) \; K(x-\xi, y-\eta, M, \omega) d\xi d\eta \tag{1}$$

where $\overline{w}(x, y)$ represents the amplitude of normal velocity or downwash at any point x, y of the surface, $\overline{\Delta p}(\xi,\eta)$ represents the amplitude of pressure at any point (ξ, η) of the surface, and $K(x-\xi, y-\eta, M, \omega)$ represents a known kernel function of the integral equation and physically is the downwash at a point (x, y) produced by a pressure pulse of unit magnitude at point (ξ, η). (See reference for derivation and reduction of the kernel function in both subsonic and supersonic flow.)

Although this integral equation may appear simple in form, there is actually little hope that it can ever be rigorously inverted except for very special plan forms and downwash conditions. One can, however, especially with high-speed computing equipment now available, obtain good approximation to the pressure distribution by methods similar to those developed for steady wings. Approximations are arrived at by simply assuming that the pressure distribution is represented by a finite series of appropriately chosen modes of pressure, where each mode is assigned an unknown coefficient. The downwash is then evaluated from this pressure representation by means of Eq. (1), and is set equal to the given downwash at a number of points on the surface equal to the number of unknown coefficients in the pressure expansion; a set of linear equations thus results which allow evaluation of the unknown coefficients.

This procedure is ideally suited for automatic digital computing machines and is hardly feasible without such machines. At the Langley Laboratory use is made of an IBM-704 computing machine and a few sample results of calculations are now presented.

The first results pertain to a rigid delta plan form of aspect ratio 4 oscillating about midchord. This particular case is chosen because there are experimentally measured results with which comparisons can be made. In Figure 1 are shown the lift and associated phase angles plotted against the reduced frequency parameter $k = \frac{b\omega}{V}$, where b is the semichord of the midsection, ω is the circular frequency, and V is stream velocity. The calculated results are seen to fit fairly well in the pattern of scatter of the measured results. A similar agreement was found in the center-of-pressure position, which is significant from the point of view of predicting flutter reliably.

Figure 2 pertains to a 60° delta oscillating in pitch about the midchord and is intended to illustrate (a) the degree of accuracy that is obtainable with the numerical handling of the kernel function mentioned above, and (b) the sizable spanwise variations that are associated with low-aspect-ratio plan forms. In this particular case, the forces could be calculated rigorously by a method of expanding the velocity potential in powers of frequency as, for example, is done in [8] and these results are the exact solution shown on the figure; the approximate solution is that obtained by numerical treatment. Very good agreement is noted. The marked spanwise variations may be noted by considering strip theory as a reference condition. Strip theory would indicate, for example, a linear variation in lift while for phase, which is very significant in flutter calculation, only constant values for both lift and moment would be noted across the span.

4. Problems of Airplane Stability in Rolling Maneuvers

After World War II, attention of the NACA was turned to research on transonic and supersonic aerodynamics. One technique for this research was the dropping of heavily weighted model airplanes from high-flying airplanes. In these tests, conducted by the Flight Research Division, it was observed that some of the models exhibited unusual longitudinal behavior while rolling. In attempting to analyze this behavior, conventional airplane stability theory was found to be inadequate because it considered the longitudinal and lateral modes separately. Previous attempts to solve the theory for combined longitudinal and lateral motion had been unsuccessful because the equations were too complicated for the computing equipment then available. In the theory presented by W. H. Phillips in reference [9], it was shown that by omitting certain terms in the equations and including others previously neglected, the equations showing the principal effects of rolling motion could be placed in a linearized form which were readily solvable by available techniques. Not only could individual cases be worked out, but general charts were presented showing the effects of wide variations in airplane design. The trends of airplane design were shown to be such that serious difficulties could be expected in future airplanes as a result of rapid rolling.

During the ensuing years, several full-scale airplanes experienced difficulties of the type predicted [10] . Though some crashes occurred, the availability of a theory to explain the difficulty provided sufficient knowledge to make safe flight tests to study the phenomenon, permitted the avoidance of critical conditions on existing designs, and allowed rapid incorporation of changes to remove the dangerous flight characteristics. The application of this knowledge may have saved the lives of many pilots. In any case, it has allowed airplanes now being built to be designed more safely with respect to this condition.

Since difficulties of this type have been experienced on full-scale airplanes, the phenomenon has been given the name "roll coupling" or "inertia coupling" and many reports have been published on this general subject, some of which are given in the list of references [11, 12] . The development of digital and analog computers capable of handling the complete equations of motion of an airplane has made it possible to calculate more precisely the transient motions of airplanes in rolling maneuvers, and has allowed inclusion of nonlinear aerodynamic characteristics peculiar to individual airplane designs. Most of these more complicated studies have verified the basic predictions of the simpler theory. This theory therefore remains a useful guide in preliminary design and in laying out more detailed flight or analytical studies on a given airplane. In addition, this theory has indicated the most satisfactory types of automatic control for avoiding roll coupling difficulties [13, 14].

Computed time histories of 360° rolls shown in Fig. 3 illustrate the effects of the roll-coupling terms.

5. Random Process Techniques in Aeronautics

The last five years have seen the rapid development and introduction of random process techniques in the solution of a variety of aeronautical problems. These techniques have had their early roots in statistical theories of turbulence, economic studies, and communication noise problems. Their application to aeronautical problems has provided deeper insight and many useful results in the studies of these problems and they are already beginning to show their effects on aircraft design. Some of the problems to which these techniques have been applied are:

a. Aircraft responses to atmospheric turbulence.

b. Airplane loads due to buffeting, noise, and runway taxiing.

c. Development of tracking and guidance systems.

d. Wind tunnel design.

e. Ocean wave and ship response problems.

The elements of random process techniques as applied to aeronautical problems are schematically illustrated in Fig. 4 for the case of a linear system exposed to a stationary random disturbance. This is the most elementary and perhaps also the most useful case.

The fundamental starting point of random process techniques is the analytic representation of the random disturbance by means of a significant statistical characterization. For the special case of stationary Gaussian random processes, this description is succinctly embodied in the single function, the power spectral density function. The power spectral density function $\varphi(\omega)$ of a random process $x(t)$ is defined by

$$\varphi_x(\omega) = \frac{2}{\pi} \int_0^{\infty} R_x(\tau) \cos \omega\tau \, d\tau \tag{2}$$

where in turn $R(\tau)$ is the autocorrelation function and is defined by

$$R_x(\tau) = \underset{T \to \infty}{\text{Limit}} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t+\tau)\, dt \tag{3}$$

The upper curve in Fig. 4 represents a power spectrum of some input disturbance, for example, atmospheric turbulence, and describes the contribution of the various frequencies to the power or energy of the disturbance $x(t)$.

A second significant aspect of random process technique is the appropriate representation of the system characteristics. For the case of a linear time-invariant system, the classical frequency response function provides the necessary and complete system representation.

The second curve illustrates a typical system frequency response function, such as the amplitude of the response to unit sinusoidal gusts at the various frequencies. The curve shown is actually for the amplitude squared of the response. The peaks in the curve normally correspond to airplane degrees of freedom or modes such as the short period or structural vibration modes.

The next important element of the technique is the input-output relations between the statistical characteristics of the disturbance and the statistical characteristics of the system response. The power spectra of the input $\varphi_x(\omega)$ and the output response $\varphi_y(\omega)$ are related simply by the following relation:

$$\varphi_y(\omega) = \varphi_x(\omega) \left| T(\omega) \right|^2 \tag{4}$$

where the bars represent the absolute value.

The power spectrum of the response may be used to derive many of the important characteristics of the actual response time history on the basis of available statistical theory.

Another useful relation exists between an input and an output and involves an extension of the concept of a power spectrum to two disturbances. This quantity is termed the cross-spectrum $\varphi_{xy}(\omega)$ which is, in turn, defined by

$$\varphi_{xy}(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} R_{xy}(\tau) e^{-i\omega\tau} dt \tag{5}$$

where, in turn, $R_{xy}(\tau)$ the cross-correlation function is defined by

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, y(t + \tau) dt \tag{6}$$

In terms of the cross-spectrum $x(t)$ and $y(t)$ are related by

$$\varphi_{xy}(\omega) = \varphi_x(\omega) T(\omega) \tag{7}$$

Note that this relation involves both the amplitude and phase of the complex quantity $T(\omega)$, whereas with Eq. (3) there is no phase information retained between input and output.

We now turn to a few examples to illustrate some of the results that have so far been obtained:

a. Effects of flexibility on airplane strain responses in rough air [15]. Figure 5 shows a comparison of measured and calculated results for the B-29 airplane frequency response functions for gust disturbances. The results on the left represent the frequency response functions obtained from flight test measurements for the strain responses at various stations. The reference curve shown represents a quasi-static reference condition and serves to provide a base for evaluating the magnitude of the flexibility effects at the various stations.

The results on the right represent the frequency response functions as derived from the airplane equations of motion. In this case, three degrees of freedom were used: vertical motion, and a first and second wing bending mode. The agreement between the measured and calculated results is seen to be quite good and serves to substantiate the adequacy of the simplified analysis employed.

b. Panel excitation from acoustic loading. Figure 6 is given to illustrate the nature of some of the results that have been obtained in studies of the behavior of structural panels under random acoustic loading [16]. The plot, which gives the variation in root-mean-square stress at a point on the panel with the square root of the input spectral intensity at the natural frequency of the panel, contains results for both a flat panel and a curved panel having a radius of 4 ft. Considering the complexity of the problem, a reasonably good agreement is found between the measured results and those predicted by the spectral technique.

c. Lateral response of airplanes to random turbulence. Random process techniques have been applied in recent years to the problem of the lateral response of an airplane to gusts. Diederich [17] derived the statistical properties of the rolling moments due to gusts on the basis of the theory of isotropic turbulence. The calculation of these effects was greatly simplified by dividing the expression for the rolling moment into a product of two functions, one of which depended only on the wing spanwise load distribution and the other on the spectrum of gust velocities. Calculations of the rolling and yawing moments on wings in random turbulence based on this theory are presented in [18].

A problem remained of accounting for the moments on the fuselage and vertical tail with a degree of accuracy similar to that afforded for the wing. A more accurate calculation of the fuselage and tail moments based on slender body theory was presented in [19].

Finally, the method of combining the gust effects on the wing and fuselage-tail combination to calculate the overall lateral response of an airplane was given in [20]. A typical roll angle response spectrum is shown in Fig. 7. This method was further simplified and placed in a form suitable for routine calculations in [21]. This report shows that the early method of assuming gust velocity distributions equivalent to rigid body motions of the airplane may be refined to provide results equivalent to the more exact method of [20]. The refinements consist in replacing some of the constant aerodynamic stability derivatives with complex quantities to account for the gust penetration effects and determining the correct relations between spectra of rolling, yawing and side gusts to yield results in agreement with the more exact analysis. This method provides a clear physical picture of the relations between various sources of lateral gust disturbances.

6. Aircraft Noise

Aircraft noise is now a major consideration in the design and operation of aircraft, mainly because of the increased noise intensity associated with the more powerful propulsion systems needed for supersonic and hypersonic flight of aircraft and missiles. The noise intensities are often sufficiently great to cause fatigue failures of the airplane structure in the vicinity of the propulsive jet, malfunctioning of electronic equipment, and interference with communications, as well as the better known physiological effects on man such as annoyance, discomfort, and hearing impairment. The results given below come from the work of Harvey H. Hubbard (NACA Langley), Newell D. Sanders (NACA Lewis) and their associates.

Fig. 8 shows the relation between acoustic power and jet stream power for cold air supersonic jets of widely varying sizes and for rocket engines ranging in thrust from about 1000 to 130,000 pounds. On the average about 0.5 percent of the energy of the jet stream is radiated as noise. This simple relationship may be useful for the prediction of the noise levels at points not too close to the source, many jet diameters distant.

Lighthill's well-known theoretical analysis [22] relates the noise of a subsonic jet to the turbulence of the jet. The analysis shows that the actual field of turbulent velocities can be represented by an equivalent field of quadrupoles which are stationary in space and fluctuating with time. This procedure is analogous to the familiar representation of flow fields as the superposition of flows from sources and vortices on a uniform flow. Quadrupoles introduce no net flow or net force and hence do not disturb the equations of conservation of mass and momentum.

The pressure p_q generated at point at a distance r from the quadrupole and at azimuth angle φ to the direction of air flow is given by the relation

$$p_q = i\rho\ C \sin\varphi \cos\varphi \left(\frac{\omega^3}{ra^2} + \frac{3i\omega^2}{r^2 a} + \frac{3\omega}{r^3} \right) e^{i\omega\left(t - \frac{r}{a}\right)} \tag{8}$$

where ρ is the air density, C the strength of the quadrupole, ω the circular frequency, a the velocity of sound, t the time.

The relation between the shear and velocities in the near field of a quadrupole and the sound pressures in the far field are known. Consequently the measured properties of the turbulent field can be used with the aid of the quadrupole relationships to compute the far sound field.

Lighthill then made some guesses concerning the character of the turbulent field of jets and, subsequently, these guesses have been substantiated with measurements by hot-wire anemometers.

These guesses (abridged) are:

(1) The intensity of turbulence, expressed as a percentage of mean jet velocity, is independent of jet velocity.

(2) The scale of turbulence is independent of jet velocity.

(3) The frequencies of the turbulent fluctuations are related to jet velocity and diameter by a Strouhal number whose value is independent of jet velocity.

As a consequence of these assumptions, Lighthill predicted that the sound power P should vary with the eighth power of the jet velocity as indicated by the following relation:

$$P = K \rho_0 A V^8/(a_0)^5 \qquad (9)$$

where K is a constant,

ρ_0 is the density of surrounding medium,

A is the cross-sectional area of nozzle, V is the mean velocity of efflux of the jet, and

a_o is the speed of sound in surrounding medium.

Experimental measurements of the sound from simple air jets, turbojet engines, and turbojet engines with afterburners have been taken and the results are shown in Fig. 9. The total sound power was obtained by integrating microphone measurements at many points in the far field surrounding the jets. The sound power in kilowatts is shown on the ordinate with a logarithmic scale. The Lighthill parameter is shown likewise on the abscissa. The data fall along a single line with a slope of unity. This result is in excellent agreement with Lighthill's prediction.

Supersonic jets generate noise both from turbulence and from shock waves. In some experiments with jets having strong shock waves, the noise from the shocks predominated. In other experiments with essentially shock-free supersonic jets, the turbulence noise predominated and the noise levels were in agreement with values predicted by the Lighthill relations.

At high jet velocities above 2800 feet per second, the power in the noise field is approaching the power in turbulence and a non-linear reaction of the sound field upon the turbulence becomes important. As a consequence the noise is less than might be expected according to the Lighthill relation and experiments with low velocity jets. The highest data point in Fig. 9 was obtained from a turbojet engine with afterburner. The point falls below the line through the other data because of this non-linear reaction.

These and other results indicate that the principal noise from a jet engine is generated by turbulence which results when the jet mixes with the surrounding air. The alleviation of jet engine noise is clearly dependent upon the modification or reduction of this turbulence.

In the far field hitherto discussed only the first term in equation (8) need be considered, whereas near the jet the other terms become large and must be considered. The pressures in the far field vary approximately as the fourth power of the velocity, corresponding to acoustic power varying as the eighth power as computed by Lighthill; in the near field the sound pressure varies approximately as the second power of the velocity. Fig. 10 shows the agreement of these predictions with sample measurements in the field of a small high-temperature jet. The near field measurements were made at two jet diameters distance, the far field at 150 diameters.

7. Effects of Viscous Interaction and Leading Edge Bluntness on a Flat Plate at Hypersonic Speeds

At least 25 years ago Prandtl discussed the effect of the boundary layer on the flow field about airfoils. The effect was in general negligible in the leading edge region at the subsonic speeds which were of interest at the time. However, when, about eight years ago, detailed studies of airfoils at Mach numbers up to 6.9 in the NACA Langley 11-inch hypersonic wind tunnel were initiated, this effect assumed immediate importance. The presence of the laminar boundary layer on flat plates was found to cause a significant increase in the pressures on the plate. The pressures were largest near the leading edge and decayed as the trailing edge was approached. Theoretical studies showed that the use of the simple concept suggested by Prandtl, that the displacement thickness be taken as the boundary of a new body, gave excellent agreement with the pressure distributions that were being measured at Mach number 6.9. Some of this work was reported in [23] and the complete experimental work in [24]. The more complete version of the theoretical work together with a detailed view of the experiments was given in [25].

At about the same time that [25] was published Lees and Probstein showed an approach to the viscous interaction problem which allowed an extraction of the parameters which governed the magnitude of the viscous effects [26]. Their approach was through the so-called "weak interaction" theory where the pressure increases caused by the displacing effect of the boundary layer are assumed to be small enough so that the boundary layer still remains essentially parabolic in shape. This correlation parameter is generally designated

$$\bar{\chi} \text{ where } \bar{\chi} \equiv M_{\infty}^{3}\sqrt{C}/\sqrt{R_{\infty}}$$

in which M_{∞} is the asymptotic or free stream Mach number, R_{∞} is the Reynolds number based on asymptotic or free stream conditions, and C is the coefficient in the linear equation for viscosity or $C = \mu_w T_{\infty}/\mu_{\infty} T_w$ where the subscript w refers to conditions at the wall.

The regime in which the pressures induced by the laminar boundary layer are so large as to affect the boundary layer thickness itself is the so-called "strong interaction" regime. This regime was treated in [25]; however, the approach was through an iteration procedure which did not reveal the basic correlating parameter. Lees [27] showed that $\bar{\chi}$ was also basically the correlating parameter for this strong interaction regime.

Until comparatively recently data to check the predictions of theory over an extended range of $\bar{\chi}$ has not been available. Much of the available data have been complicated by an extraneous effect of leading edge bluntness of which more will be said later. Kendall at the California Institute of Technology [28] for a Mach number of 5.8 and Bertram at NACA-Langley [29] at Mach 9.6 have recently provided data which are believed to be free of leading edge thickness effects. These results show theory can be relied upon to provide estimates of the viscous interaction effect.

Recently additional experiments have been made by Bertram and his associates in the 11-inch hypersonic tunnel to investigate the effect of angle of attack and to extend the range of $\bar{\chi}$. Previously unreported data from this experiment are given in Fig. 11. (In the figure $C_e = \mu_e T_2 / \mu_2 T_e$ where e refers to insulated plate conditions, and 2 to the asymptotic or boundary-layer-free plate conditions at angle of attack.) The open symbols are the original data selected at times when the temperature gradient shown in Fig. 12 existed on the plate. (The curve in Fig. 12

is a representation of the equation fitted to the temperatures so that the Chapman-Rubesin theory could be applied as was done in [29]). Using the Chapman-Rubesin theory the data were modified to insulated plate conditions shown as the darkened symbols on Fig. 11. The pressure difference parameter (ordinate) and $\overline{\chi}$ (abscissa) are all based on the asymptotic conditions on the plate or the conditions that would prevail on the plate if the boundary layer had not been present. Good agreement is obtained with the Lees first order strong interaction theory over the entire range shown. This figure presents data to the largest values of $\overline{\chi}$ yet obtained in air and is a critical check of the theory.

In hypersonic flow large pressure gradients can be induced on a flat plate from another source besides boundary layer displacement effects. This source is the finite thickness of leading edge of plates which in the ordinary case, considering their dimensions, might be thought of as sharp. Again this effect was encountered quite early in hypersonic experience. This pressure gradient is a result of the large entropy gradient in the flow field about blunt noses in hypersonic flow. In [30] a model was proposed for treating this problem by characteristics theory. The limiting case that could be treated by simple characteristics theory, the sonic wedge, was proposed as an approximation to an actual blunt leading edge. Recently Cheng and Pallone [31] and Lees and Kubota [32] have given the proper correlating parameters for this blunt nose phenomenon. Their work stems from the so-called "blast wave" theory in which the two-dimensional blunt leading edge is taken as the origin of an explosion in which the energy released is proportional to the leading edge drag. The theoretically predicted correlation parameter has been found to correlate the results from characteristics theory utilizing the sonic wedge as the blunt leading edge [33]. Fig. 13 is an extension of Fig. 1(a) of [33] ($\gamma = 7/5$) and shows that the parameter suggested by two-dimensional blast wave theory does indeed correlate the more exact characteristics solutions. This result lends considerable justification to the theory. In addition the correlation curve may be used to predict blunt leading edge pressures. The region very close to the nose which deviates from the correlation curve is the region where blast wave theory is not expected to apply. The blast wave theory parameter in the abscissa is $(x/t)/(M_\infty^3 k)$ where k is the nose drag coefficient (based on nose forward projected area).

In practical hypersonic cases the bluntness effect and the viscous effect will exist together. No theoretical treatment of the combined effects has yet been developed. In [30] a simple linear combination of viscous-effect theory and sonic-wedge bluntness effect theory was shown to be in reasonable agreement with experiment. Bogdonoff [34, 35] has also obtained results in helium at a Mach number of about 11 which show the same trend.

Recently Henderson in the 2-inch helium jet at Langley has obtained data in this region of combined effects in the Mach number range of 17 to 23 at zero angle of attack under insulated plate conditions. Fig. 14 presents data at his highest and lowest Mach numbers for various leading edge Reynolds numbers (based on free stream conditions and leading edge thickness). The pure viscous theory shown on the plot is obtained by the method of [25] utilizing the Lees-Probstein [26] $\overline{\chi}$ parameter. The other theoretical lines result from simple addition of the increment in pressure predicted by blunt leading edge theory using a correlation curve of the characteristics theory for helium similar to the one for air presented in Fig. 13. Thus there is assumed to be no interaction between the two effects. The agreement is better than one would anticipate. The apparent disagreement at the low values of R_t is random and thus does not appear to be basic.

As indicated by the foregoing discussion much progress has been made in our knowledge of the viscous and bluntness effects on the wall pressures. Extension of these results to the prediction of local skin friction, heat transfer, and to three-dimensional effects is obviously essential.

8. Combustion Instability of Rockets

In the development of combustion chambers for rockets instabilities of combustion have been encountered which often result in destruction of the chamber by failure under vibratory stress or burnout. If the disturbances are predominantly of low frequency, the phenomenon is called chugging. A much higher frequency mode, usually accompanied by greatly increased heat transfer rate or burnout, is called screeching or screaming. The origin and nature of these instabilities are not fully understood. It appears probable that there is no single cause.

In some cases of chugging the instability results from coupling between fuel flow and combustion chamber pressure associated with certain time delays. If the chamber pressure is momentarily lowered, the propellant flow will begin to increase. But there is a time lag because of the inertia of the fluid in the propellant supply line. The increased fuel flow increases the combustion rate but there is a second time delay caused by the time needed for injection, mixing, vaporizing, ignition, and combustion. Since the combustion chamber has a finite volume, additional time is required to fill the chamber with burned gas. There is thus an overshoot in chamber pressure and if the damping is insufficient, an instability may result. Such a case is studied both theoretically and experimentally in [36]. Fig. 15 shows a typical comparison of theory and experiment. Two chambers were used at an average chamber pressure of 270 pounds per square inch absolute using white fuming nitric acid and commercial n-heptane as propellants. The parameter L^* is the combustion volume divided by throat area. The study shows that a sufficiently high injection pressure drop eliminates chugging from this cause.

Screeching or screaming is commonly found associated with strong acoustic oscillations of the longitudinal, radial, or tangential type. These affect the mixing process as shown by the experiments and analyses of Moore, Mickelsen and their co-workers [37, 38]. Experimental results [39] obtained with the NACA high speed camera verified that the wave traveled within the chamber as predicted. Available experimental results have since been correlated, as shown in Fig. 16, to predict the frequency of screaming engines. Here the characteristic velocity C* is equal to the product of chamber pressure, throat area, and acceleration of gravity divided by the rate of propellant flow.

In some other cases of instability, a longitudinal shock oscillation in the manner of the V-1 pulse jet engine has been noted.

9. Motion and Aerodynamic Heating of Missiles Entering the Earth's Atmosphere at High Supersonic Speeds

A problem attacked in the spirit of Prandtl by H. Julian Allen of NACA Ames Laboratory is that of minimizing the aerodynamic heating of a missile entering the atmosphere at high speeds. The analysis was made in 1952, reported in a classified document, and published in 1957 [40].

Allen's analysis assumes an exponential variation of air density with altitude, that variations in drag coefficient may be neglected, and that the retardation is so large that the gravity term in the equations of motion may be neglected. On these assumptions the maximum deceleration depends only on the entry speed and flight path angle, occurring when the missile has decelerated to 61 percent of the entry speed. The altitude for maximum deceleration y_1 depends only on the physical characteristics of the missile and flight-path angle but not on the entry speed, being given by the equation

$$y_1 = \frac{1}{\beta} \ln \left\{ \left(C_D \rho_0 A \right) \Big/ \left(\beta m \sin \theta_E \right) \right\} \qquad (10)$$

where β is the parameter in the exponential variation of air density i e $\rho = \rho_o e^{-\beta y}$; C_D is the drag coefficient; ρ_o the density at a reference altitude, both ρ_o and β being chosen to give the best fit over the range of altitudes for which the deceleration is large; A the reference area for drag coefficient evaluation; m the mass of the missile, and θ_E the entry angle.

Reynolds' analogy is used to determine the heat transfer, the local heat transfer coefficient being assumed to be proportional to the local skin-friction coefficient. The rate of transfer to the whole surface with altitude may be shown to be dependent on the parameter appearing in equation (10) and on a new parameter $\left(C_{f'} \; S\rho_o V_E 2/4 \sin \theta_E\right)$ where $C_{f'}$ is an equivalent friction coefficient obtained by integration over the whole wetted area S. The maximum rate of heat transfer occurs at the same altitude as maximum deceleration. The total heat input to the body at impact, obtained by integration is found to be equal to $1/2\left(C_{f'} \; S/C_D A\right)$ times the total reduction in kinetic energy of the missile from entry to impact.

For the case where the retardation is very great, the impact speed is low and the quantity $1/2\left(C_{f'} \; S/C_D A\right)$ represents the fraction of the total kinetic energy which is transferred to the missile as heat; the remainder is transferred to the air. If the skin friction drag is a small part of the total, the heating problem becomes much easier. This analysis led Allen to the blunt nose concept as a means of greatly reducing aerodynamic heating, a concept fully demonstrated by later experiments.

Allen's analysis is a simplified one, not exact, but gives a good overall view of the problem.

A comparative analysis along similar lines of the performance and heating of long-range hypervelocity vehicles of the ballistic, skip, and glide types is given in [41] .

10. Reentry of Satellites in Planetary Atmospheres

Chapman [42] has generalized the analysis of the reentry motion and heating to lifting and non-lifting vehicles in trajectories of the type indicated in Fig. 17. This analysis was stimulated by current interest in satellites and aimed at a general understanding of the reentry problem.

The methods employed here also have a close analogy with Prandtl's boundary-layer analysis. The complete equations of motion, like the complete Navier-Stokes equations, are too complex to permit a general analytical solution. A solution is first attempted only for a model atmosphere (exponential density-altitude relationship), just as a solution to the viscous flow equations was first attempted only for a model fluid of constant properties. By physical arguments, certain terms in the complete equations are reasoned to be relatively small and are disregarded. In the case of the motion equations, for example, the acceleration term in polar coordinates uv/r (where u is the horizontal velocity component of the vehicle, v the vertical velocity component, and r the distance from the planet center) is disregarded compared to du/dt. This procedure yields results valid for small flight-path angles, excluding about the first percent of velocity reduction; by analogy Prandtl's procedure yields results valid for thin boundary layers, excluding about the first percent or so of plate length. Also by analogy, the resulting equations are further simplified by reduction to a single dimensionless equation through a mathematical transformation. In the case of the motion equations the transformed variable is

$$Z \equiv \frac{g\sqrt{r/\beta}}{2\left(\frac{W}{C_D A}\right)}\,\bar{u}\,\rho_\infty \qquad (11)$$

where $\bar{u} \equiv u/\sqrt{gr}$ is the ratio of the horizontal velocity relative to the orbital velocity, β is the constant in the atmosphere approximation $\rho_\infty/\rho_o = e^{-\beta y}$, g is the acceleration of gravity, W is the weight, C_D the drag coefficient, and A the reference area of the vehicle. The resulting differential equation can be given simple physical significance:

$$\underbrace{\bar{u}\frac{d^2Z}{d\bar{u}^2}}_{\substack{\text{vertical}\\\text{acceleration}}} - \underbrace{\frac{dZ}{d\bar{u}} - \frac{Z}{\bar{u}}}_{\substack{\text{vertical}\\\text{drag}\\\text{component}}} = \underbrace{\frac{1-\bar{u}^2}{\bar{u}Z}}_{\substack{\text{gravity}\\\text{minus}\\\text{centrifugal}\\\text{force}}} - \underbrace{\sqrt{\beta r}\,\frac{L}{D}}_{\substack{\text{lift}\\\text{force}}} \qquad (12)$$

Here L/D is the lift-drag ratio. The basic equation is non-linear, just as the resulting boundary-layer equation is non-linear. The initial conditions for reentry from decaying orbits are simple. At $\bar{u} = 1$; $Z = 0$, $\frac{dZ}{d\bar{u}} = 0$.

One solution is required for each value of the lift-drag parameter $\sqrt{\beta r}\,\frac{L}{D}$ just as one solution to the boundary-layer equation is required for each pressure gradient. Each solution to the non-linear equation is universal in the sense that, once obtained, it is applicable to any planetary atmosphere irrespective of the physical characteristics of the vehicle; each solution to the boundary-layer equation similarly is applicable irrespective of the fluid mediums or the physical dimensions of the surface. For example, one solution to equation (12) for $L/D = 0$ provides a universal solution for all non-lifting bodies in the same sense that the Blasius solution for $dp/dx = 0$ provides a universal solution for all flat plates.

The above equation ties in very nicely with Allen's solution for ballistic reentry. Allen neglected the gravity force, centrifugal force, and considered non-lifting vehicles only. These considerations correspond to setting all terms on the right side of the above equation equal to zero. The resulting terms on the left side are linear and then admit to a solution $Z \sim \bar{u}\, \ln\, \bar{u}$ which yields results identical to those of Allen's solution.

Typical results of the analysis are shown in Figs. 18, 19, and 20. Fig. 18 shows the maximum deceleration encountered in the slow spiraling in of a satellite orbit about several planets. Figs. 19 and 20 show the effect of lift/drag ratio on deceleration and laminar heating rate for orbital decay of earth satellites.

It will be some time in the future before we can add experimental points to these curves.

11. Dynamic Stability of Reentry Vehicles

Allen has analyzed [43] the oscillating motion of a ballistic missile which enters the atmosphere with angular misalignment with respect to the flight path. This is again an analysis based on simplifying assumptions which preserve the essential physical factors and thus permits a grasp of the overall stability problem for the non-lifting reentry body.

The lifting case has recently been studied by Tobak and Allen [44], the specific example of a skip trajectory being treated in detail. Friedrich and Dore [45] recently gave an approximate solution for the envelope curve of the oscillation of angle of attack, if the location of the first maximum is known.

Tobak and Allen developed an approximate solution of the angle of attack equation from whose form the main characteristics of the motion can be deduced. It turns out that the characteristic mode of oscillation is described by a Bessel function rather than a trigonometric function. The physical reason is clear. As a vehicle descends through the atmosphere the rapidly increasing atmospheric density causes the aerodynamic restoring moment to act in the manner of a stiffening spring. Thus the oscillation must reduce in amplitude and increase in frequency, a behavior described by one of the Bessel functions. These solutions, illustrated in Figs. 21 and 22, apply to any trajectory for which the direction of flight does not change rapidly with altitude.

12. High-Speed Free-Molecule and Slip Flow

The life expectancy of an orbiting satellite can be measured in terms of the parasitic drag forces acting upon it. Although small in terms of aerodynamic forces in the ordinary sense these drag forces act over very long periods of time and extract enough energy from the satellite to cause orbit decay and eventual reentry into the lower atmosphere. The temperature of the satellite skin, and therefore the temperature of the internal structure, inhabitant, and equipment are determined by a balance of the heat energy at the satellite surface. These energies consist of radiation from the sun, re-radiation to outer space and the energy delivered to the satellite by molecular impact. While a satellite is in equilibrium orbit the ambient atmospheric pressure is low enough so that the mean-molecular-free path is large compared to the body dimension and the satellite is flying in the free-molecule regime. (A 20-inch-diameter sphere is in free-molecule flow above an altitude of 390,000 feet.) As the orbit decays and the flight altitude decreases the satellite enters the slip-flow regime. An investigation has therefore been undertaken to determine the drag and heat-transfer characteristics of bodies in high-speed free-molecule and slip flow.

A theory has been developed which predicts the heat-transfer rates, temperatures, and drag forces on a body of arbitrary shape in high-speed free-molecule flow. This theory was developed by the application of the ideas of classical kinetic theory in order to calculate the mass and energy flux striking an element of surface area. An energy balance was made at the solid surface of the body with the assumption that the gas molecules lose all velocity at impact with the surface and are re-emitted from the surface with an energy equivalent to a gas having a Maxwellian speed distribution in equilibrium with the body temperature. To obtain drag forces on or heat transfer to an arbitrary body these expressions were integrated over the body surface.

A comparison has been made between this theory and the results of low-density wind-tunnel tests performed to determine the drag and temperature rise characteristic of a transverse circular cylinder [46]. The tests were conducted under conditions where the Knudsen number of the flow (ratio of the mean-free-molecule path to the cylinder radius) varied between 4 and 185.

The measured values of the cylinder temperature confirmed the salient point of the heat-transfer analysis which was the prediction that a cylinder would attain a temperature higher than the stagnation temperature of the stream. These results are shown in Fig. 23. On this figure are plotted the ratio of the model temperature to the free-stream static temperature as a function of the molecular speed ratio (ratio of the stream mass velocity to the most probable molecular speed). Two curves are shown on this figure. The solid-line curve is the result of the free-molecule-flow analysis and the dash-line curve is the temperature a cylinder would attain in continuum flow. It can be seen that a cylinder in free-molecule flow has attained a temperature which is considerably higher than would be attained under similar conditions in a continuum flow. The tests points agree reasonably well with the free-molecule flow theory for molecular-speed ratios greater than about 1.6.

The drag forces on a transverse cylinder were also measured for conditions of free-molecule flow. These results are shown in Fig. 24 wherein the drag coefficient is plotted as a function of the molecular speed ratio. Good agreement was attained between the theoretical and the experimental values of the drag coefficient. As predicted, theoretically, the drag coefficient was independent of Knudsen number and dependent primarily upon the molecular-speed ratio.

In the regime of flow characterized by mean-molecular free paths of the order of a boundary-layer thickness, the slip-flow regime, adequate theory has not been developed; however, tests in the low-density tunnel have been conducted over a range of Knudsen numbers which represent free-molecule flow, slip-flow, and continuum flow over transverse cylinders [47] . In this series of tests the equilibrium temperature and the heat-transfer coefficients of transverse cylinders were measured.

It was found from these tests that fully developed free-molecule flow occurred at Knudsen numbers greater than two. No clear division of the continuum and slip-flow regime could be discerned from these tests. For Knudsen numbers corresponding to free-molecule flow ($K > 2$) the heat transfer could be correlated by the theoretical expression,

$$Nu = 0.184 \, Re \tag{13}$$

where Nu is the Nusselt number and Re is the Reynolds number.

In the region where the flow was definitely continuum the heat transfer could be correlated by the expression

$$Nu = 0.57 \, Re^{1/2} \tag{14}$$

In the intermediate region, slip flow, the best fit of the heat-transfer data resulted in the following expression

$$Nu = 0.132 \, Re^{0.73} \tag{15}$$

It appears from the results of these investigations that theory is adequate to predict heating and forces on bodies in free-molecule flow if the atmosphere in which the body is flying is known and if the energy exchange process at the body surface can be specified. In the slip-flow regime, however, more experimental work is needed.

13. Continued Studies of Transition from Laminar to Turbulent Flow

In September 1955 I presented a review [48] of research work on the problem of transition in the boundary layer, a phenomenon first noted by Prandtl. Work continues on this problem both within NACA laboratories and at numerous other institutions. My own interest in this problem began many years ago at the National Bureau of Standards, and from the beginning the work there was sponsored by NACA.

The recent work of the NBS group at subsonic speeds under the direction of Schubauer will shortly be published [49]. A definite and reproducible progression of events has been found by which the amplified

Tollmien-Schlichting waves lead to turbulent flow. The essential features are as follows:

1. The wave motions become strongly three-dimensional prior to transition.

2. Accompanying the wave motion there is an energy concentrating mechanism giving rise to a transfer of wave energy from one spanwise position to a neighboring one.

3. There is evidence of longitudinal vortices, i.e., with axes parallel to the flow direction.

4. The breakdown of laminar flow occurs in the regions in which the energy is concentrated.

5. The initial breakdown of laminar flow occurs in the outer region of the boundary layer and is of such small extent that it may be loosely described as point-like.

6. The local regions of breakdown are the initial stages in the development of turbulent spots.

7. Intermittent separation is not present in the transition of a laminar boundary layer with zero pressure gradient.

The whole picture is consistent with Görtler's suggestion [50] that the curvature of flow in the amplified Tollmien-Schlichting waves gives rise to a three-dimensional instability in the form of vortices of the type studied by him.

Further studies have been made of the effect of roughness on transition. I have recently analyzed the existing data on the effect of single two-dimensional roughness elements on transition in air streams of varying turbulence [51]. It appears that the effect of a single two-dimensional roughness element with given ratio of height k to boundary-layer displacement-thickness $\delta_k{}^*$ is the same as the effect of an "equivalent" turbulence level equal to $4.4\left(k/\delta_k{}^*\right)^3$ percent.

Evidence continues to accumulate that three-dimensional roughness elements sufficiently submerged in the boundary layer do not cause transition until the Reynolds number based on the velocity at the top of the roughness and the roughness height exceeds a critical value, at which point turbulence is generated by the element.

Fig. 25 shows the results of von Doenhoff and Horton [52] obtained on an NACA $65_{(215)}$-114 airfoil which has a favorable pressure gradient (falling pressure) over the forward half of the chord. The critical roughness Reynolds number based on maximum rather than nominal roughness height is about 600 except in the forward part of the boundary layer where the roughness protruded nearly through the boundary layer.

Braslow has shown [53] that the critical roughness Reynolds number is approximately the same on a cone at supersonic speed as on the airfoil at subsonic speed when the value of kinematic viscosity is computed for the temperature and pressure prevailing in the boundary layer at the top of the roughness element. Fig. 26 shows Braslow's results. Because of the effects of Mach number on the kinematic viscosity the effect of a given roughness element decreases with Mach number. Cooling the surface however decreases the kinematic viscosity and may increase the Reynolds number enough to produce transition from existing roughness elements.

14. Conclusion

These few examples of current problems must suffice to illustrate the main thesis of this lecture -- that the spirit of Prandtl is still alive today. I hope that I have convinced you that NACA scientists are occupied with ideas as well as with gigantic wind tunnels and high speed electronic computing machines. We too try to honor Prandtl by advancing knowledge in the sciences he established and by utilizing his methods in the study of the new problems of today. In the name of aeronautical research workers, engineers, and designers in the United States and of the NACA in particular, I salute the memory of Ludwig Prandtl.

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16. Figure Legends

Fig. 1. Variation of Lift for a Delta Wing of Aspect Ratio 4 Oscillating about Midchord.

Fig. 2. Comparison of Exact and Approximate Calculations of Spanwise Variation of Lift and Pitching Moment for a Delta Wing of Aspect Ratio 2.31 at a Mach Number of $\sqrt{2}$ and Reduced Frequency Parameter 0.375.

Fig. 3. Effect of Roll Coupling Terms on Time Histories of 360° Rolls.

Fig. 4. Input-Output Relation for Power Spectra.

Fig. 5. Strain Frequency Response Function for B-29 Airplane.

Fig. 6. Stress in Panel under Acoustic Loading as Function of Input Spectral Intensity at Natural Frequency of Panel.

Fig. 7. Power Spectral Density of Roll Angle Response in Flight Through Continuous Atmospheric Turbulence.

Fig. 8. Relation of Acoustical Power to Jet Power.

Fig. 9. Relation of Acoustical Power to Lighthill Parameter.

Fig. 10. Variation of Sound Pressure with Jet Velocity for Near and Far Field.

Fig. 11. Wall Temperature Distribution on Flat Plate in Hypersonic Flow with Boundary-Layer Induced Pressures.

Fig. 12. Boundary-Layer Induced Pressures on Flat Plate in Hypersonic Flow at Mach Number of 9.6.

Fig. 25. Critical Reynolds Number of Roughness Element to Produce Transition on an Airfoil at Subsonic Speed as Function of Chordwise Location.

Fig. 26. Critical Reynolds Number of Roughness Element to Produce Transition on a Cone at Supersonic Speed as Function of Distance from Cone Apex.